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Wandzura-Wilczek-type relations of ρ-meson wave functions

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Abstract: We give the geometric Wandzura-Wilczek-type relations between the meson wave functions of dynamical twist by means of the meson wave functions of geometric twist. We discuss the difference between geometric and dynamical Wandzura-Wilczek-type relations. Additionally, the interrelations between the different twist notations of meson wave functions are discussed.

Keywords: QCD, Sum Rules.

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Contents

1. Introduction

In a recent paper [1], I have introduced the (two-particle) *ρ-meson wave functions* which are related to nonlocal LC-operators of different geometric twist [2, 3]. This twist decomposition of nonlocal operators is based on the notion of *geometric* twist = mass dimension — (Lorentz) spin, $\tau = d - j$, originally introduced by Gross and Treiman [4]. I found eight meson wave functions of geometric twist.

On the other hand, Ball et al. used the notion of *dynamical twist t*, which was introduced by Jaffe and Ji [5] by counting powers Q^{2-t}, for the classification of (two-particle) vector meson wave functions and vector meson distribution amplitudes, respectively [6]–[8]. They classified the meson wave functions which correspond to the independent tensor structure of the matrix elements of bilocal quark-antiquark operators. One key ingredient in their approach was the use of QCD equations of motion in order to obtain dynamical Wandzura-Wilczek-type relations for wave functions that are not dynamically independent. The pion wave functions were investigated in similar way [9]–[11]. A systematic study of Wandzura-Wilczek-type relations for the forward case is given in ref. [12].

In the framework of geometric twist, it is possible to investigate the interrelations and the mismatch between the different twist definitions of meson wave functions as a kind of group theoretical relations. By means of this relations, we are able to obtain geometric Wandzura-Wilczek-type relations between the the dynamical twist function which differ from the dynamical Wandzura-Wilczek-type relations.

Twist t	2 O(1)	3 $\mathrm{O}(1/Q)$	4 $\mathrm{O}(1/Q^2)$
$e_\parallel$	$\hat{\phi}_\parallel \equiv \hat{\Phi}^{(2)}$	$\underline{\hat{h}_\parallel^{(t)}} = \hat{\Phi}^{(3)} + F_t(\hat{\Phi}^{(2)}, \hat{\Phi}^{(3)})$, $\underline{\hat{h}_\parallel^{(s)}} \equiv \hat{\Upsilon}^{(3)}$	$\hat{g}_3 = \hat{\Phi}^{(4)} + F_g(\hat{\Phi}^{(2)}, \hat{\Phi}^{(3)}, \hat{\Phi}^{(4)})$
$e_\perp$	$\underline{\hat{\phi}_\perp} \equiv \hat{\Psi}^{(2)}$	$\hat{g}_\perp^{(v)} = \hat{\Psi}^{(3)} + F_v(\hat{\Psi}^{(2)}, \hat{\Psi}^{(3)})$, $\hat{g}_\perp^{(a)} \equiv \hat{\Xi}^{(3)}$	$\underline{\hat{h}_3} = \hat{\Psi}^{(4)} + F_h(\hat{\Psi}^{(2)}, \hat{\Psi}^{(3)}, \hat{\Psi}^{(4)})$

Table 1: Spin, dynamical twist and chiral classification of the ρ-meson distribution amplitudes.

2. The dynamical twist ρ-meson wave functions

Usually, the meson light-cone distributions are defined as the vacuum-to-meson matrix elements of quark-antiquark nonlocal gauge invariant operators on the light-cone. In this way, Ball et al. [6]–[8] found eight independent two-particle distributions. The classification of these dynamical twist ρ-meson wave functions with respect to spin, dynamical twist, chirality and the relations to geometrical twist (see section 3) are summarized in table 1. One distribution amplitude was obtained for longitudinally ($e_\parallel$) and transversely ($e_\perp$) polarized ρ-mesons of twist-2 and twist-4, respectively. Whereas, the number of twist-3 distribution amplitudes is doubled for each polarization. The higher twist distribution amplitudes contribute to a hard exclusive amplitude with additional powers of $1/Q$ compared to the leading twist-2 ones. The underlined distribution amplitudes are chiral-odd, the others chiral-even.

By using the Lorentz decomposition of the matrix elements of relevant operators in analogy to nucleon structure functions, the explicit definitions of the chiral-even ρ-distributions are:[1]

$$
\begin{aligned}
\langle 0|\bar{u}(\tilde{x})\gamma_\alpha U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle = f_\rho m_\rho \Bigg[& p_\alpha \frac{e^{(\lambda)}\tilde{x}}{\tilde{x}P}\int_{-1}^{1}\mathrm{d}\xi\, \mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{\phi}_\parallel(\xi,\mu^2) + \\
& + e_{\perp\alpha}^{(\lambda)}\int_{-1}^{1}\mathrm{d}\xi\, \mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{g}_\perp^{(v)}(\xi,\mu^2) - \frac{1}{2}\times \\
& \times \tilde{x}_\alpha \frac{e^{(\lambda)}\tilde{x}}{(\tilde{x}P)^2} m_\rho^2 \int_{-1}^{1}\mathrm{d}\xi\, \mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{g}_3(\xi,\mu^2)\Bigg]
\end{aligned}
\tag{2.1}
$$

and

$$
\begin{aligned}
\langle 0|\bar{u}(\tilde{x})\gamma_\alpha\gamma_5 U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle = & \frac{1}{2}\left(f_\rho - f_\rho^{\mathrm{T}}\frac{m_u+m_d}{m_\rho}\right) m_\rho \epsilon_\alpha{}^{\beta\mu\nu} e_{\perp\beta}^{(\lambda)} p_\mu \tilde{x}_\nu \times \\
& \times \int_{-1}^{1}\mathrm{d}\xi\, \mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{g}_\perp^{(a)}(\xi,\mu^2)\,,
\end{aligned}
\tag{2.2}
$$

[1] Here I am using the symmetric wave function $\hat{\phi}(\xi)$ (see also [13]) instead of $\phi(u)$ because the corresponding integral relations will be easier.

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while the chiral-odd distributions are defined as

$$\begin{aligned}
&\langle 0|\bar{u}(\tilde{x})\sigma_{\alpha\beta}U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle = \\
&= \mathrm{i} f_\rho^{\mathrm{T}}\Big[(e^{(\lambda)}_{\perp\alpha}p_\beta - e^{(\lambda)}_{\perp\beta}p_\alpha)\int_{-1}^{1}\mathrm{d}\xi\, \mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{\phi}_\perp(\xi,\mu^2) + (p_\alpha\tilde{x}_\beta - p_\beta\tilde{x}_\alpha)\frac{e^{(\lambda)}\tilde{x}}{(\tilde{x}P)^2}m_\rho^2\,\times \\
&\quad\times\int_{-1}^{1}\mathrm{d}\xi\,\mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{h}_\parallel^{(t)}(\xi,\mu^2) + \frac{1}{2}(e^{(\lambda)}_{\perp\alpha}\tilde{x}_\beta - e^{(\lambda)}_{\perp\beta}\tilde{x}_\alpha)\frac{m_\rho^2}{\tilde{x}P}\int_{-1}^{1}\mathrm{d}\xi\,\mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{h}_3(\xi,\mu^2)\Big]
\end{aligned} \tag{2.3}$$

and

$$\begin{aligned}
\langle 0|\bar{u}(\tilde{x})U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle &= -\mathrm{i}\left(f_\rho^{\mathrm{T}} - f_\rho\frac{m_u+m_d}{m_\rho}\right)(e^{(\lambda)}\tilde{x})m_\rho^2\,\times \\
&\quad\times\int_{-1}^{1}\mathrm{d}\xi\,\mathrm{e}^{\mathrm{i}\xi(\tilde{x}P)}\hat{h}_\parallel^{(s)}(\xi,\mu^2)\,.
\end{aligned} \tag{2.4}$$

In the above definitions we used the two light-cone vectors

$$\tilde{x}_\alpha = x_\alpha - \frac{P_\alpha}{m_\rho^2}\Big((xP) - \sqrt{(xP)^2 - x^2m_\rho^2}\Big)\,, \qquad p_\alpha = P_\alpha - \frac{1}{2}\tilde{x}_\alpha\frac{m_\rho^2}{\tilde{x}P}\,, \tag{2.5}$$

with $p^2 = 0$, $\tilde{x}^2 = 0$ and $p\cdot\tilde{x} = P\cdot\tilde{x}$. Here P_α is the ρ-meson momentum vector, so that $P^2 = m_\rho^2$, $e^{(\lambda)}\cdot e^{(\lambda)} = -1$, $P\cdot e^{(\lambda)} = 0$, m_ρ denotes the ρ-meson mass. The polarization vector $e^{(\lambda)}_\alpha$ can be decomposed into projections onto the two light-like vectors and the orthogonal plane:

$$e^{(\lambda)}_\alpha = p_\alpha\frac{e^{(\lambda)}\tilde{x}}{\tilde{x}P} - \frac{1}{2}\tilde{x}_\alpha\frac{e^{(\lambda)}\tilde{x}}{(\tilde{x}P)^2}m_\rho^2 + e^{(\lambda)}_{\perp\alpha}\,. \tag{2.6}$$

The distribution amplitudes are dimensionless functions of ξ and describe the probability amplitudes to find the ρ in a state with minimal number of constituents (quark and antiquark) which carry momentum fractions ξ. The nonlocal operators are renormalized at scale μ, so that the distribution amplitudes depend on μ as well. This dependence can be calculated in perturbative QCD (from now on we suppress μ^2).

The vector and tensor decay constants f_ρ and f_ρ^{T} are defined as usually as

$$\langle 0|\bar{u}(0)\gamma_\alpha d(0)|\rho(P,\lambda)\rangle = f_\rho m_\rho e^{(\lambda)}_\alpha\,, \tag{2.7}$$

$$\langle 0|\bar{u}(0)\sigma_{\alpha\beta}d(0)|\rho(P,\lambda)\rangle = \mathrm{i}f_\rho^{\mathrm{T}}(e^{(\lambda)}_\alpha P_\beta - e^{(\lambda)}_\beta P_\alpha)\,. \tag{2.8}$$

All eight distributions $\hat{\phi} = \{\hat{\phi}_\parallel, \hat{\phi}_\perp, \hat{g}_\perp^{(v)}, \hat{g}_\perp^{(a)}, \hat{h}_\parallel^{(t)}, \hat{h}_\parallel^{(s)}, \hat{h}_3, \hat{g}_3\}$ are normalized as

$$\int_{-1}^{1}\mathrm{d}\xi\,\hat{\phi}(\xi) = 1\,. \tag{2.9}$$

The wave function moments are given as

$$\phi_n = \int_{-1}^{1} \mathrm{d}\xi\, \xi^n \hat{\phi}(\xi) \equiv \int_0^1 \mathrm{d}u\, \xi^n \phi(u), \qquad \xi = 2u - 1 \tag{2.10}$$

with both types of meson wave functions

$$\hat{\phi}(\xi) = \frac{1}{2}\,\phi\left(\frac{\xi+1}{2}\right), \qquad \phi(u) = 2\hat{\phi}(2u-1)\,. \tag{2.11}$$

Eq. (2.10) means, having information about the wave function moments, one can reconstruct the wave function itself.

3. The geometric twist ρ-meson wave functions and relations to dynamical twist meson wave functions

Reproducing results of [1], we define the ρ-meson wave functions of geometric twist. By means of operators of geometric twist [1], the chiral-even ρ-distributions with definite (geometric) twist are:

$$\begin{aligned}
\langle 0|\bar{u}(\tilde{x})\gamma_\alpha U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle &= f_\rho m_\rho \bigg[p_\alpha \frac{e^{(\lambda)}\tilde{x}}{\tilde{x}P} \int_{-1}^{1} \mathrm{d}\xi\, \hat{\Phi}^{(2)}(\xi) e_0(\mathrm{i}\zeta\xi) + e^{(\lambda)}_{\perp\alpha} \times \\
&\times \int_{-1}^{1} \mathrm{d}\xi \Big\{ \hat{\Phi}^{(2)}(\xi) e_1(\mathrm{i}\zeta\xi) + \hat{\Phi}^{(3)}(\xi)[e_0(\mathrm{i}\zeta\xi) - e_1(\mathrm{i}\zeta\xi)] \Big\} - \frac{1}{2}\tilde{x}_\alpha \frac{e^{(\lambda)}\tilde{x}}{(\tilde{x}P)^2} m_\rho^2 \times \\
&\times \int_{-1}^{1} \mathrm{d}\xi \Big\{ \hat{\Phi}^{(4)}(\xi) \Big[e_0(\mathrm{i}\zeta\xi) - 3e_1(\mathrm{i}\zeta\xi) + 2\int_0^1 \mathrm{d}t\, e_1(\mathrm{i}\zeta\xi t) \Big] - \\
&- \hat{\Phi}^{(2)}(\xi) \Big[e_1(\mathrm{i}\zeta\xi) - 2\int_0^1 \mathrm{d}t\, e_1(\mathrm{i}\zeta\xi t) \Big] + 4\hat{\Phi}^{(3)} \Big[e_1(\mathrm{i}\zeta\xi) - \int_0^1 \mathrm{d}t\, e_1(\mathrm{i}\zeta\xi t) \Big] \Big\} \bigg]
\end{aligned} \tag{3.1}$$

and

$$\begin{aligned}
\langle 0|\bar{u}(\tilde{x})\gamma_\alpha\gamma_5 U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle &= \frac{1}{2}\Big(f_\rho - f_\rho^{\mathrm{T}} \frac{m_u + m_d}{m_\rho} \Big) m_\rho \epsilon_\alpha{}^{\beta\mu\nu} e^{(\lambda)}_{\perp\beta} p_\mu \tilde{x}_\nu \times \\
&\times \int_{-1}^{1} \mathrm{d}\xi\, \hat{\Xi}^{(3)}(\xi) e_0(\mathrm{i}\zeta\xi)\,,
\end{aligned} \tag{3.2}$$

while the chiral-odd distributions are defined as

$$\begin{aligned}
&\langle 0|\bar{u}(\tilde{x})\sigma_{\alpha\beta} U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle = \\
&= \mathrm{i} f_\rho^{\mathrm{T}} \bigg[2e^{(\lambda)}_{\perp[\alpha}p_{\beta]} \int_{-1}^{1} \mathrm{d}\xi \hat{\Psi}^{(2)}(\xi) e_0(\mathrm{i}\zeta\xi) + 2\tilde{x}_{[\alpha}p_{\beta]} \frac{e^{(\lambda)}\tilde{x}}{(\tilde{x}P)^2} m_\rho^2 \times \\
&\qquad \times \int_{-1}^{1} \mathrm{d}\xi \Big\{ 2\hat{\Psi}^{(2)}(\xi) e_2(\mathrm{i}\zeta\xi) - \hat{\Psi}^{(3)}(\xi)[e_0(\mathrm{i}\zeta\xi) + 2e_2(\mathrm{i}\zeta\xi)] \Big\} - \\
&\qquad - \tilde{x}_{[\alpha} e^{(\lambda)}_{\beta]\perp} \frac{m_\rho^2}{\tilde{x}P} \int_{-1}^{1} \mathrm{d}\xi \Big\{ 2\hat{\Psi}^{(2)}(\xi)[e_1(\mathrm{i}\zeta\xi) + e_2(\mathrm{i}\zeta\xi)] - \\
&\qquad\qquad - \hat{\Psi}^{(3)}(\xi)[1 + 2e_2(\mathrm{i}\zeta\xi)] + \hat{\Psi}^{(4)}(\xi)[1 + e_0(\mathrm{i}\zeta\xi) - 2e_1(\mathrm{i}\zeta\xi)] \Big\} \bigg]
\end{aligned} \tag{3.3}$$

and

$$\langle 0|\bar{u}(\tilde{x})U(\tilde{x},-\tilde{x})d(-\tilde{x})|\rho(P,\lambda)\rangle = -\mathrm{i}\Big(f_\rho^{\mathrm{T}} - f_\rho\frac{m_u+m_d}{m_\rho}\Big)(e^{(\lambda)}\tilde{x})m_\rho^2 \times$$
$$\times \int_{-1}^{1}\mathrm{d}\xi\, \hat{\Upsilon}^{(3)}(\xi)e_0(\mathrm{i}\zeta\xi)\,. \tag{3.4}$$

Here we used the following "truncated exponentials" (with $\zeta = (\tilde{x}P)$)

$$\begin{aligned}
e_0(\mathrm{i}\zeta\xi) &= \mathrm{e}^{\mathrm{i}\zeta\xi}\,,\\
e_1(\mathrm{i}\zeta\xi) &= \int_0^1 \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\zeta\xi t} = \frac{\mathrm{e}^{\mathrm{i}\zeta\xi}-1}{\mathrm{i}\zeta\xi}\,,\\
e_{n+1}(\mathrm{i}\zeta\xi) &= \frac{(-1)^n}{n!}\int_0^1 \mathrm{d}t\,t^n\,\mathrm{e}^{\mathrm{i}\zeta\xi t}\,.
\end{aligned} \tag{3.5}$$

Comparing these expressions (3.1)–(3.4) with the meson wave functions of dynamical twist (2.1)–(2.4), we observe that it is necessary to re-express the truncated exponentials and perform appropriate variable transformations. After such manipulations we obtain the following relations, which allow to reveal the interrelations between the different twist definitions of meson wave functions:

$$\hat{\phi}_{\|}(\xi) \equiv \hat{\Phi}^{(2)}(\xi)\,, \tag{3.6}$$

$$\hat{g}_{\perp}^{(v)}(\xi) = \hat{\Phi}^{(3)}(\xi) + \int_{\xi}^{\mathrm{sign}(\xi)}\frac{\mathrm{d}\tau}{\tau}\Big(\hat{\Phi}^{(2)} - \hat{\Phi}^{(3)}\Big)(\tau)\,, \tag{3.7}$$

$$\hat{g}_3(\xi) = \hat{\Phi}^{(4)}(\xi) - \int_{\xi}^{\mathrm{sign}(\xi)}\frac{\mathrm{d}\tau}{\tau}\Big\{\Big(\hat{\Phi}^{(2)} - 4\hat{\Phi}^{(3)} + 3\hat{\Phi}^{(4)}\Big)(\tau) +$$
$$+ 2\ln\Big(\frac{\xi}{\tau}\Big)\Big(\hat{\Phi}^{(2)} - 2\hat{\Phi}^{(3)} + \hat{\Phi}^{(4)}\Big)(\tau)\Big\}\,, \tag{3.8}$$

$$\hat{g}_{\perp}^{(a)}(\xi) \equiv \hat{\Xi}^{(3)}(\xi)\,, \tag{3.9}$$

$$\hat{\phi}_{\perp}(\xi) \equiv \hat{\Psi}^{(2)}(\xi)\,, \tag{3.10}$$

$$\hat{h}_{\|}^{(t)}(\xi) = \hat{\Psi}^{(3)}(\xi) + 2\xi\int_{\xi}^{\mathrm{sign}(\xi)}\frac{\mathrm{d}\tau}{\tau^2}\Big(\hat{\Psi}^{(2)} - \hat{\Psi}^{(3)}\Big)(\tau)\,, \tag{3.11}$$

$$\hat{h}_3(\xi) = \hat{\Psi}^{(4)}(\xi) + \int_{\xi}^{\mathrm{sign}(\xi)}\frac{\mathrm{d}\tau}{\tau}\Big\{2\Big(\hat{\Psi}^{(2)} - \hat{\Psi}^{(4)}\Big)(\tau) - 2\frac{\xi}{\tau}\Big(\hat{\Psi}^{(2)} - \hat{\Psi}^{(3)}\Big)(\tau) -$$
$$-\,\delta\Big(\frac{\xi}{\tau}\Big)\Big(\hat{\Psi}^{(3)} - \hat{\Psi}^{(4)}\Big)(\tau)\Big\}, \tag{3.12}$$

$$\hat{h}_{\|}^{(s)}(\xi) \equiv \hat{\Upsilon}^{(3)}(\xi)\,. \tag{3.13}$$

Obviously, both decompositions coincide in the leading terms, e.g. the meson wave functions $\hat{\phi}_{\|}(\xi)$ and $\hat{\phi}_{\perp}(\xi)$ are genuine geometric twist-2 and $\hat{g}_{\perp}^{(a)}(\xi)$ and $\hat{h}_{\|}^{(s)}(\xi)$ are genuine geometric twist-3 functions. But the different twist definitions of meson wave functions differ at higher twist. For instance, the meson wave functions $\hat{g}_{\perp}^{(v)}(\xi)$ and $\hat{h}_{\|}^{(t)}(\xi)$ with dynamical twist $t = 3$ contain contributions with geometrical twist $\tau = 2$ and 3. Additionally, dynamical twist $t = 4$ meson wave functions $\hat{g}_3(\xi)$ and $\hat{h}_3(\xi)$ contain contributions with geometrical twist $\tau = 2$, 3 as well as 4.

The conventional wave functions can be written in terms of the new ρ-meson wave functions. The nontrivial relations are:

$$\hat\Phi^{(3)}(\xi) = \hat g_\perp^{(v)}(\xi) + \frac{1}{\xi}\int_\xi^{\mathrm{sign}(\xi)} \mathrm{d}\tau (\hat g_\perp^{(v)} - \hat\phi_\parallel)\,(\tau)\,, \tag{3.14}$$

$$\begin{aligned}\hat\Phi^{(4)}(\xi) &= \hat g_3(\xi) + \frac{1}{\xi^2}\int_\xi^{\mathrm{sign}(\xi)} \mathrm{d}\tau\,\tau (3\hat g_3 - 4\hat g_\perp^{(v)} + \hat\phi_\parallel)\,(\tau) + \\ &\quad + \frac{1}{\xi^2}\int_\xi^{\mathrm{sign}(\xi)} \mathrm{d}\tau\,\tau \Big(1-\frac{\xi}{\tau}\Big) (\hat g_3 - 4\hat g_\perp^{(v)} + 3\hat\phi_\parallel)\,(\tau)\,,\end{aligned} \tag{3.15}$$

$$\hat\Psi^{(3)}(\xi) = \hat h_\parallel^{(t)}(\xi) + \frac{2}{\xi}\int_\xi^{\mathrm{sign}(\xi)} \mathrm{d}\tau (\hat h_\parallel^{(t)} - \hat\phi_\perp)\,(\tau)\,, \tag{3.16}$$

$$\begin{aligned}\hat\Psi^{(4)}(\xi) &= \hat h_3(\xi) + \frac{2}{\xi^2}\int_\xi^{\mathrm{sign}(\xi)} \mathrm{d}\tau\,\tau (\hat h_3 - \hat h_\parallel^{(t)})\,(\tau) - \\ &\quad - \frac{2}{\xi^2}\int_\xi^{\mathrm{sign}(\xi)} \mathrm{d}\tau\,\tau \Big(1-\frac{\xi}{\tau}\Big) (\hat h_\parallel^{(t)} - \hat\phi_\perp)\,(\tau)\,.\end{aligned} \tag{3.17}$$

The relations between the moments may be read off from eqs. (3.1)–(3.4) as follows:

$$\begin{aligned}
\phi_{\parallel n} &\equiv \Phi_n^{(2)}\,, \\
g_{\perp n}^{(v)} &= \Phi_n^{(3)} + \frac{1}{n+1}\Big(\Phi_n^{(2)} - \Phi_n^{(3)}\Big)\,, \\
g_{3n} &= \Phi_n^{(4)} - \frac{1}{n+1}\Big(\Phi_n^{(2)} - 4\Phi_n^{(3)} + 3\Phi_n^{(4)}\Big) + \frac{2}{(n+1)^2}\Big(\Phi_n^{(2)} - 2\Phi_n^{(3)} + \Phi_n^{(4)}\Big)\,, \\
g_{\perp n}^{(a)} &\equiv \Xi_n^{(3)}\,, \\
\phi_{\perp n} &\equiv \Psi_n^{(2)}\,, \\
h_{\parallel n}^{(t)} &= \Psi_n^{(3)} + \frac{2}{n+2}\Big(\Psi_n^{(2)} - \Psi_n^{(3)}\Big)\,, \\
h_{3n} &= \Psi_n^{(4)} + \frac{2}{n+1}\Big(\Psi_n^{(2)} - \Psi_n^{(4)}\Big) - \frac{2}{n+2}\Big(\Psi_n^{(2)} - \Psi_n^{(3)}\Big) - \delta_{n0}\Big(\Psi_n^{(3)} - \Psi_n^{(4)}\Big)\,, \\
h_{\parallel n}^{(s)} &\equiv \Upsilon_n^{(3)}.
\end{aligned} \tag{3.18}$$

In terms of the moments the relations between old and new wave functions may be easily inverted; for the wave functions itself the expression of the new wave functions through the old ones is more involved. The inverse relations are:

$$\begin{aligned}
\Phi_n^{(3)} &= g_{\perp n}^{(v)} + \frac{1}{n}\Big(g_{\perp n}^{(v)} - \phi_{\parallel n}\Big)\,, \qquad n>0 \\
\Phi_n^{(4)} &= g_{3n} + \frac{1}{n-1}\Big(3g_{3n} - 4g_{\perp n}^{(v)} + \phi_{\parallel n}\Big) + \frac{1}{n(n-1)}\Big(g_{3n} - 4g_{\perp n}^{(v)} + 3\phi_{\parallel n}\Big)\,, \quad n>1 \\
\Psi_n^{(3)} &= h_{\parallel n}^{(t)} + \frac{2}{n}\Big(h_{\parallel n}^{(t)} - \phi_{\perp n}\Big)\,, \quad n>0 \\
\Psi_n^{(4)} &= h_{3n} + \frac{2}{n-1}\Big(h_{3n} - h_{\parallel n}^{(t)}\Big) - \frac{2}{n(n-1)}\Big(h_{\parallel n}^{(t)} - \phi_{\perp n}\Big)\,, \quad n>1\,.
\end{aligned} \tag{3.19}$$

The relations between the dynamical and geometrical twist meson wave functions show that the dynamical wave functions can be used to determine the new geometrical ones and vice versa. Obviously, the same holds for their moments. In principle, this allows to determine, e.g., the new wave functions from the experimental data if these are known for the conventional ones. Thus, all of the physical and experimental correlations obtained by Ball et al. [6, 7, 8], e.g. the asymptotic wave functions, can be used for the geometrical twist wave functions.

4. Wandzura-Wilczek-type relations

This section is devoted to the general discussion of geometric Wandzura-Wilczek-type relations between the conventional wave functions which we have obtained by means of the wave functions with a definite geometric twist. These geometric Wandzura-Wilczek-type relations are independent from the QCD field equations and the corresponding operator relations. The mismatch between the definition of dynamical and geometric twist gives rise to relations of geometric Wandzura-Wilczek-type which show that the dynamical twist functions contain various parts of different geometric twist. Consequently, a genuine geometric twist wave function do not give rise to any geometric Wandzura-Wilczek-type relation. Thereby, we obtain new (geometric) Wandzura-Wilczek-type relations and sum rules for the meson wave functions of dynamical twist.

We use the notations $\hat{g}_\perp^{(v)}(\xi) = \hat{g}_\perp^{(v),\mathrm{tw2}}(\xi) + \hat{g}_\perp^{(v),\mathrm{tw3}}(\xi)$, where $\hat{g}_\perp^{(v),\mathrm{tw2}}(\xi)$ is the genuine twist-2 and $\hat{g}_\perp^{(v),\mathrm{tw3}}(\xi)$ the genuine twist-3 part of $\hat{g}_\perp^{(v)}(\xi)$. Substituting (3.6) into (3.7), we get

$$\hat{g}_\perp^{(v),\mathrm{tw2}}(\xi) = \int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau}\, \hat{\phi}_\parallel(\tau)\,, \tag{4.1}$$

$$\hat{g}_\perp^{(v),\mathrm{tw3}}(\xi) = \hat{g}_\perp^{(v)}(\xi) - \int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau}\, \hat{\phi}_\parallel(\tau)\,, \tag{4.2}$$

where (4.1) is really the analogue of the Wandzura-Wilczek relation for the twist-2 part [14] because they argued that the geometric twist-3 contribution should be small. On the other hand, eq. (4.2) is the Wandzura-Wilczek-type relation of geometric twist-3 and gives the information how much is the physical contribution of the geometric twist-3 operator in this non-forward process. These relations show that the transverse distribution $\hat{g}_\perp^{(v)}(\xi)$ is related to the longitudinal distribution $\hat{\phi}_\parallel(\xi)$. Obviously, this analogy between the Wandzura-Wilczek relations of parton distribution functions and of meson wave functions is not surprising because the operator structures are the same and the ρ-meson polarization vector formally substitutes the nucleon spin vector in the Lorentz structures. The generalization of the geometric Wandzura-Wilczek relation (4.1) for non-forward distribution amplitudes was discussed in ref. [15]. Let me note that the Wandzura-Wilczek relations which are obtained in refs. [16, 6, 7] for $g_\perp^{(v),\mathrm{tw2}}(u)$ are significantly different (see also [17]). The reason is that they used equations of motion operator relations in order to isolate the dynamical twist-3 part. Therefore, their Wandzura-Wilczek relations are dynamical relations (dynamical Wandzura-Wilczek-type relations). The corresponding geometric Wandzura-Wilczek-type relations for the moments are:

$$g_{\perp n}^{(v),\mathrm{tw2}} = \frac{1}{n+1}\,\phi_{\|n}\,, \tag{4.3}$$

$$g_{\perp n}^{(v),\mathrm{tw3}} = g_{\perp n}^{(v)} - \frac{1}{n+1}\,\phi_{\|n}\,, \qquad n>0\,. \tag{4.4}$$

Note that eq. (4.3) was earlier obtained in ref. [6]. The reason for this agreement between the twist-2 part of the dynamical and geometric Wandzura-Wilczek relation is that the dynamical and geometric twist-2 meson wave function coincides. Obviously, from the relation (4.3) for $n=0$ and $n=1$ the following sum rules follow:

$$\int_{-1}^{1} \mathrm{d}\xi\, \hat{g}_\perp^{(v)}(\xi) = \int_{-1}^{1} \mathrm{d}\xi\, \hat{\phi}_\|(\xi) = 1, \qquad \int_{-1}^{1} \mathrm{d}\xi\, \xi \hat{g}_\perp^{(v),\mathrm{tw2}}(\xi) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}\xi\, \xi \hat{\phi}_\|(\xi). \tag{4.5}$$

Using the formulas (3.6), (3.7), (3.8) and (3.14), we obtain the integral relations for the function $\hat{g}_3(\xi) = \hat{g}_3^{\mathrm{tw2}}(\xi) + \hat{g}_3^{\mathrm{tw3}}(\xi) + \hat{g}_3^{\mathrm{tw4}}(\xi)$ as

$$\hat{g}_3^{\mathrm{tw2}}(\xi) = -\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau}\Big\{\hat{\phi}_\|(\tau) + 2\ln\Big(\frac{\xi}{\tau}\Big)\hat{\phi}_\|(\tau)\Big\}, \tag{4.6}$$

$$\begin{aligned}\hat{g}_3^{\mathrm{tw3}}(\xi) = {} & 4\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau}\,\hat{g}_\perp^{(v)}(\tau) + 4\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau^2}\int_\tau^{\mathrm{sign}(\tau)} \mathrm{d}\omega(\hat{g}_\perp^{(v)} - \hat{\phi}_\|)(\omega) + \\ & +4\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau}\ln\Big(\frac{\xi}{\tau}\Big)\hat{g}_\perp^{(v)}(\tau) + 4\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau^2}\ln\Big(\frac{\xi}{\tau}\Big)\int_\tau^{\mathrm{sign}(\tau)} \mathrm{d}\omega(\hat{g}_\perp^{(v)} - \hat{\phi}_\|)(\omega)\,,\end{aligned} \tag{4.7}$$

$$\begin{aligned}\hat{g}_3^{\mathrm{tw4}}(\xi) = {} & \hat{g}_3(\xi) - \int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau}(4\hat{g}_\perp^{(v)} - \hat{\phi}_\|)(\tau) - 4\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau^2}\int_\tau^{\mathrm{sign}(v)} \mathrm{d}\omega(\hat{g}_\perp^{(v)} - \hat{\phi}_\|)(\omega) - \\ & -2\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau}\ln\Big(\frac{\xi}{\tau}\Big)(2\hat{g}_\perp^{(v)} - \hat{\phi}_\|)(\tau) - 4\int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau^2}\ln\Big(\frac{\xi}{\tau}\Big) \times \\ & \times \int_\tau^{\mathrm{sign}(\tau)} \mathrm{d}\omega(\hat{g}_\perp^{(v)} - \hat{\phi}_\|)(\omega)\,.\end{aligned} \tag{4.8}$$

Due to the fact that $\hat{g}_3(\xi)$ contains twist-2, twist-3 as well as twist-4, we have obtained three integral relations. For example, eq. (4.6) demonstrates that the twist-2 part $\hat{g}_3^{\mathrm{tw2}}(\xi)$ can be expressed in terms of the twist-2 function $\hat{\phi}_\|(\xi)$. Additionally, the twist-3 part $\hat{g}_3^{\mathrm{tw3}}(\xi)$ is given in eq. (4.7) in terms of the functions $\hat{\phi}_\|(\xi)$ and $\hat{g}_\perp^{(v)}(\xi)$. Eq. (4.8) expresses how can be obtained the twist-4 part $\hat{g}_3^{\mathrm{tw4}}(\xi)$ from $\hat{g}_3(\xi)$. For the wave function moments we obtain:

$$g_{3n}^{\rm tw2} = -\frac{1}{n+1}\,\phi_{\|n} + \frac{2}{(n+1)^2}\,\phi_{\|n}, \tag{4.9}$$

$$g_{3n}^{\rm tw3} = \frac{4}{n+1}\,g_{\perp n}^{(v)} + \frac{4}{(n+1)n}\Big(g_{\perp n}^{(v)} - \phi_{\|n}\Big) - \frac{4}{(n+1)^2}\,g_{\perp n}^{(v)} - \frac{4}{(n+1)^2 n}\Big(g_{\perp n}^{(v)} - \phi_{\|n}\Big)\,, \quad n > 0 \tag{4.10}$$

$$g_{3n}^{\rm tw4} = g_{3n} - \frac{1}{n+1}\Big(4g_{\perp n}^{(v)} - \phi_{\|n}\Big) - \frac{4}{(n+1)n}\Big(g_{\perp n}^{(v)} - \phi_{\|n}\Big) + \frac{2}{(n+1)^2}\Big(2g_{\perp n}^{(v)} - \phi_{\|n}\Big) + + \frac{4}{(n+1)^2 n}\Big(g_{\perp n}^{(v)} - \phi_{\|n}\Big), \qquad n > 1\,. \tag{4.11}$$

For $n = 0, 1$ in (4.9) we find the sum rules

$$\int_{-1}^{1} {\rm d}\xi\, \hat g_3(\xi) = \int_{-1}^{1} {\rm d}\xi\, \hat\phi_{\|}(\xi) = 1\,, \qquad \int_{-1}^{1} {\rm d}\xi\, \xi \hat g_3^{\rm tw2}(\xi) = 0\,, \tag{4.12}$$

as well as for $n = 1$ in (4.10)

$$\int_{-1}^{1} {\rm d}\xi\, \xi \hat g_3^{\rm tw3}(\xi) = \int_{-1}^{1} {\rm d}\xi\, \xi (2\hat g_\perp^{(v)} - \hat\phi_{\|})(\xi). \tag{4.13}$$

Let us now discuss the geometric Wandzura-Wilczek-type relations for the chiral-odd meson wave functions. If we substitute (3.10) into (3.11) and using $\hat h_{\|}^{(t)}(\xi) = \hat h_{\|}^{(t),\rm tw2}(\xi) + \hat h_{\|}^{(t),\rm tw3}(\xi)$ we derive

$$\hat h_{\|}^{(t),\rm tw2}(\xi) = 2\xi \int_{\xi}^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau^2}\,\hat\phi_\perp(\tau)\,, \tag{4.14}$$

$$\hat h_{\|}^{(t),\rm tw3}(\xi) = \hat h_{\|}^{(t)}(\xi) - 2\xi \int_{\xi}^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau^2}\,\hat\phi_\perp(\tau)\,, \tag{4.15}$$

where (4.14) is the analogues of the twist-2 Wandzura-Wilczek-type relation of the nucleon structure function $h_L(z)$ which was obtained by Jaffe and Ji [5]. These Wandzura Wilczek-type relations point out that the longitudinal distribution $\hat h_{\|}^{(t)}(\xi)$ is related to the transverse distribution $\hat\phi_\perp(\xi)$. Dynamical Wandzura-Wilczek relations were obtained in ref. [7]. Obviously, (4.15) is the corresponding twist-3 relation. The relations for the moments read:

$$h_{\|n}^{(t),\rm tw2} = \frac{2}{n+2}\,\phi_{\perp n}\,, \tag{4.16}$$

$$h_{\|n}^{(t),\rm tw3} = h_{\|n}^{(t)} - \frac{2}{n+2}\,\phi_{\perp n}\,, \qquad n > 0\,. \tag{4.17}$$

For $n = 0, 1$ in (4.16) we observe the following sum rules

$$\int_{-1}^{1} {\rm d}\xi\, \hat h_{\|}^{(t)}(\xi) = \int_{-1}^{1} {\rm d}\xi\, \hat\phi_\perp(\xi) = 1, \qquad \int_{-1}^{1} {\rm d}\xi\, \xi \hat h_{\|}^{(t),\rm tw2}(\xi) = \frac{2}{3}\int_{-1}^{1} {\rm d}\xi\, \xi\hat\phi_\perp(\xi). \tag{4.18}$$

Now, using the formulas (3.10), (3.11), (3.12) and (3.16), we obtain the integral relations for the function with $\hat{h}_3(\xi) = \hat{h}_3^{\rm tw2}(\xi) + \hat{h}_3^{\rm tw3}(\xi) + \hat{h}_3^{\rm tw4}(\xi)$ as

$$\hat{h}_3^{\rm tw2}(\xi) = 2\int_\xi^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau}\,\hat{\phi}_\perp(\tau) - 2\xi\int_\xi^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau^2}\,\hat{\phi}_\perp(\tau)\,, \tag{4.19}$$

$$\hat{h}_3^{\rm tw3}(\xi) = 2\xi\int_\xi^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau^2}\,\hat{h}_\parallel^{(t)}(\tau) + 4\xi\int_\xi^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau^3}\int_\tau^{{\rm sign}(\tau)} {\rm d}\omega(\hat{h}_\parallel^{(t)} - \hat{\phi}_\perp)(\omega)\,, \tag{4.20}$$

$$\hat{h}_3^{\rm tw4}(\xi) = \hat{h}_3(\xi) - 2\int_\xi^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau}\,\hat{\phi}_\perp(\tau) - 2\xi\int_\xi^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau^2}(\hat{h}_\parallel^{(t)} - \hat{\phi}_\perp)(\tau) -$$
$$-4\xi\int_\xi^{{\rm sign}(\xi)} \frac{{\rm d}\tau}{\tau^3}\int_\tau^{{\rm sign}(\tau)} {\rm d}\omega(\hat{h}_\parallel^{(t)} - \hat{\phi}_\perp)(\omega) \tag{4.21}$$

and for the moments

$$h_{3n}^{\rm tw2} = \frac{2}{n+1}\,\phi_{\perp n} - \frac{2}{n+2}\,\phi_{\perp n}\,, \tag{4.22}$$

$$h_{3n}^{\rm tw3} = \frac{2}{n+2}\,h_{\parallel n}^{(t)} + \frac{4}{(n+2)n}\Big(h_{\parallel n}^{(t)} - \phi_{\perp n}\Big)\,, \qquad n > 0 \tag{4.23}$$

$$h_{3n}^{\rm tw4} = h_{3n} - \frac{2}{n+1}\,\phi_{\perp n} - \frac{2}{n+2}\Big(h_{\parallel n}^{(t)} - \phi_{\perp n}\Big) - \frac{4}{(n+2)n}\Big(h_{\parallel n}^{(t)} - \phi_{\perp n}\Big)\,,\ n > 1. \tag{4.24}$$

Thus, eq. (4.19) means that the twist-2 part $\hat{h}_3^{\rm tw2}(\xi)$ can be expressed in terms of the twist-2 function $\hat{\phi}_\perp(\xi)$. Additionally, the twist-3 part $\hat{h}_3^{\rm tw3}(\xi)$ is given in eq. (4.20) in terms of the functions $\hat{\phi}_\perp(\xi)$ and $\hat{h}_\parallel^{(t)}(\xi)$. Eq. (4.21) gives the subtraction rule in order to obtain the twist-4 part $\hat{h}_3^{\rm tw4}(\xi)$ from the original function $\hat{h}_3(\xi)$. Obviously, the same is valid for the corresponding wave function moments.

For $n = 0, 1$ in (4.22) we observe the following sum rules

$$\int_{-1}^1 {\rm d}\xi\,\hat{h}_3(\xi) = \int_{-1}^1 {\rm d}\xi\,\hat{\phi}_\perp(\xi) = 1, \qquad \int_{-1}^1 {\rm d}\xi\,\xi\hat{h}_3^{\rm tw2}(\xi) = \frac{1}{3}\int_{-1}^1 {\rm d}\xi\,\xi\hat{\phi}_\perp(\xi)\,, \tag{4.25}$$

and for $n = 1$ in (4.23)

$$\int_{-1}^1 {\rm d}\xi\,\xi\hat{h}_3^{\rm tw3}(\xi) = \frac{1}{3}\int_{-1}^1 {\rm d}\xi\,\xi(6\hat{h}_\parallel^{(t)}(\xi) - 4\hat{\phi}_\perp(\xi))\,. \tag{4.26}$$

Because the functions $\hat{g}_\perp^{(a)}(\xi)$ and $\hat{h}_\parallel^{(s)}(\xi)$ are pure geometric as well as dynamical twist-3 functions, there is no mismatch between the dynamical and geometric twist and no geometric Wandzura-Wilczek-type relation occurs. However, dynamical Wandzura-Wilczek relations for $\hat{g}_\perp^{(a)}(\xi)$ and $\hat{h}_\parallel^{(s)}(\xi)$ were obtained by Ball et al. [6, 7] by using the QCD equations of motion. Here I am discussing the relation for $g_{\perp n}^{(a)}$. Neglecting quark masses and three-particle quark-antiquark-gluon operators of twist-3 one recovers the operator relation (see [18])

$$[\bar{u}(\tilde{x})\gamma_\alpha\gamma_5 d(-\tilde{x})]^{\rm tw3} = {\rm i}\,\epsilon_\alpha^{\ \beta\mu\nu}\tilde{x}_\beta\int_0^1 {\rm d}u\,u\,\partial_\mu[\bar{u}(u\tilde{x})\gamma_\nu d(-u\tilde{x})] + \cdots\,, \tag{4.27}$$

where $\hat{\partial}_\mu$ is the so-called total derivative which translates the expansion point y:

$$\hat{\partial}_\mu[\bar{u}(u\tilde{x})\gamma_\nu d(-u\tilde{x})] \equiv \frac{\partial}{\partial y^\mu}[\bar{u}(y+u\tilde{x})\gamma_\nu d(y-u\tilde{x})]|_{y\to 0}. \qquad (4.28)$$

The total derivative on the r.h.s. of Eq. (4.27) is calculated as

$$\begin{aligned}\hat{\partial}_\mu[\bar{u}(u\tilde{x})\gamma_\nu d(-u\tilde{x})] &= \bar{u}(u\tilde{x})(\overleftarrow{D}_\mu + \overrightarrow{D}_\mu)\gamma_\nu d(-u\tilde{x}) - \\ &\quad -\mathrm{i}\int_{-u}^{u} \mathrm{d}v\, \bar{u}(u\tilde{x})\tilde{x}^\rho F_{\rho\mu}(v\tilde{x})\gamma_\nu d(-u\tilde{x})\,, \end{aligned} \qquad (4.29)$$

where $F_{\mu\nu}$ is the gluon field strength. Obviously, the second operator on the r.h.s. of eq. (4.29) is a Shuryak-Vainshtein type operator having minimal twist-3. The other operator on the r.h.s. of eq. (4.29) has, in general, twist-2, 3 and 4. But, after multiplying by the Levi-Civita tensor $\epsilon_{\alpha\beta\mu\nu}$ only the twist-3 part survives and gives contributions which are not small. Therefore, the operator identity (4.27) is a genuine twist-3 relation. The twist-3 matrix element of the first operator on the r.h.s. of eq. (4.29) is given by

$$\begin{aligned}\mathrm{i}\,\epsilon_\alpha{}^{\beta\mu\nu}\tilde{x}_\beta\langle 0|\bar{u}(\tilde{x})(\overleftarrow{D}_\mu + \overrightarrow{D}_\mu)\gamma_\nu d(-\tilde{x})|\rho(P,\lambda)\rangle &= f_\rho m_\rho \epsilon_\alpha{}^{\beta\mu\nu} e^{(\lambda)}_{\perp\beta} p_\mu \tilde{x}_\nu \times \\ &\quad\times \int_{-1}^{1} \mathrm{d}\xi\, \hat{\Omega}^{(3)}(\xi) e_0(\mathrm{i}\zeta\xi)\,, \end{aligned} \qquad (4.30)$$

where $\hat{\Omega}^{(3)}(\xi)$ is the corresponding meson wave function of twist-3.

Taking the matrix element of the operators on both sides of eq. (4.27), using eqs. (3.2), (3.9), and (4.30), and neglecting quark masses and trilocal operators, one obtains a geometric relation between the meson wave functions $\hat{g}^{(a)}_\perp(\xi)$ and $\hat{\Omega}^{(3)}(\xi)$ imposed by the QCD equations of motion as

$$\hat{g}^{(a)}_\perp(\xi) = 2\xi \int_\xi^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau^2}\,\hat{\Omega}^{(3)}(\tau)\,, \qquad (4.31)$$

and for the meson wave function moments

$$g^{(a)}_{\perp n} = \frac{2}{n+2}\,\Omega^{(3)}_n\,, \qquad (4.32)$$

where $\Omega^{(3)}_n$ are the geometric twist-3 wave function moments. This relation (4.32) reproduces the fact that the moments $g^{(a)}_{\perp n}$ have genuine geometric twist-3 (see eq. (3.18)) and no geometric twist-2 contributions from the total derivative come into the game.

But this is not what Ball et al. [6, 7] did. They have calculated the r.h.s. of eq. (4.27) by means of

$$\hat{\partial}_\mu\langle 0|\bar{u}(\tilde{x})\gamma_\nu d(-\tilde{x})|\rho(P,\lambda)\rangle \qquad (4.33)$$

and eq. (2.1). Eventually, they obtained the dynamical Wandzura-Wilczek-type relation [6]

$$\hat{g}_{\perp}^{(a)}(\xi) = 2\xi \int_{\xi}^{\mathrm{sign}(\xi)} \frac{\mathrm{d}\tau}{\tau^2}\, \hat{g}_{\perp}^{(v)}(\tau), \tag{4.34}$$

and for the moments

$$g_{\perp n}^{(a)} = \frac{2}{n+2}\, g_{\perp n}^{(v)}. \tag{4.35}$$

Obviously, eqs. (4.34) and (4.35) are relations between equal dynamical twist. But, the l.h.s. of these relations has geometric twist-3 and the r.h.s. has geometric twist-2 and twist-3. Therefore, a dynamical Wandzura-Wilczek relation is a relation for equal dynamical twist in contrast to geometric Wandzura-Wilczek relations which are relations for equal geometric twist. Note that the Wandzura-Wilczek-like relations used in refs. [19] are dynamical ones.

Similar arguments are valid for all QCD equations of motion operator relations used in ref. [7]. Thus, from the group theoretical point of view, it is a bit misleading to claim that an additional geometric twist-2 contribution is produced by geometric twist-3 operators with a total derivative and that the QCD equations of motion give a relation between distribution functions of different geometric twist in exclusive processes and off-forward scattering. From this point of view, the geometric twist is a proper concept for classifying non-forward inclusive or general exclusive matrix-elements.

5. Conclusions

Extending an earlier study [1], we discussed the model-independent classification of meson wave functions with respect to geometric twist. We gave the relations between the geometric twist and Ball and Braun's dynamical twist wave functions. These relations demonstrate the interrelations between the different twist definitions. Consequently, these relations are "transformation rules" between the dynamical and geometric twist wave functions.

The main results of this letter are geometric Wandzura-Wilczek-type relations between the dynamical twist distributions which we have obtained by means of our "transformation rules". The reason of these geometric Wandzura-Wilczek-type relations is the mismatch of the dynamical twist with respect to geometric twist. Because we have not used the QCD equations of motion, the geometric Wandzura-Wilczek-type relations are based on a no-dynamical level and are model-independent. Additionally, I have discussed the difference between dynamical and geometric Wandzura-Wilczek-type relations.

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